



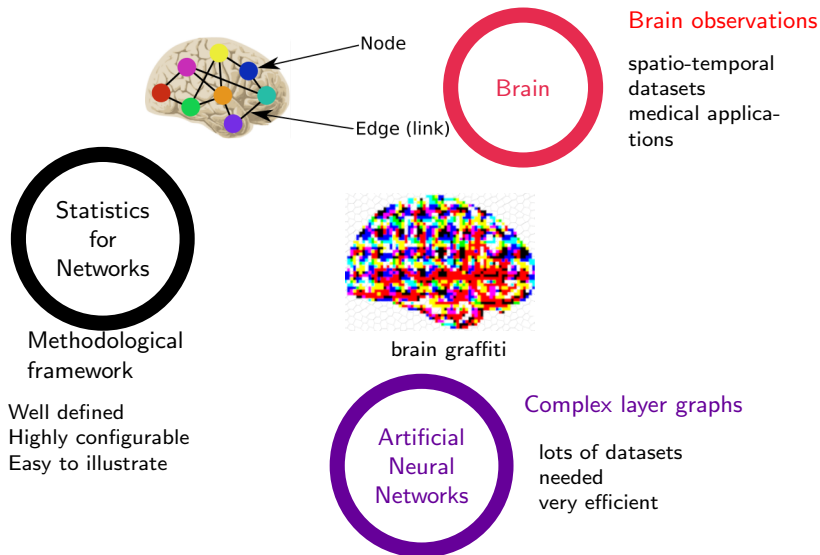
Graphing the brain: Perspectives on graph theory for the understanding of neural networks and the brain

Sophie Achard

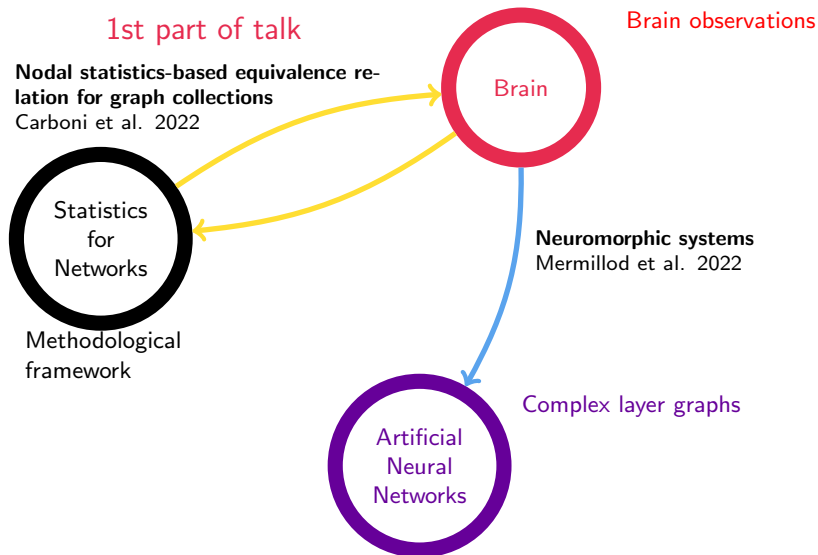
sophie.achard@univ-grenoble-alpes.fr



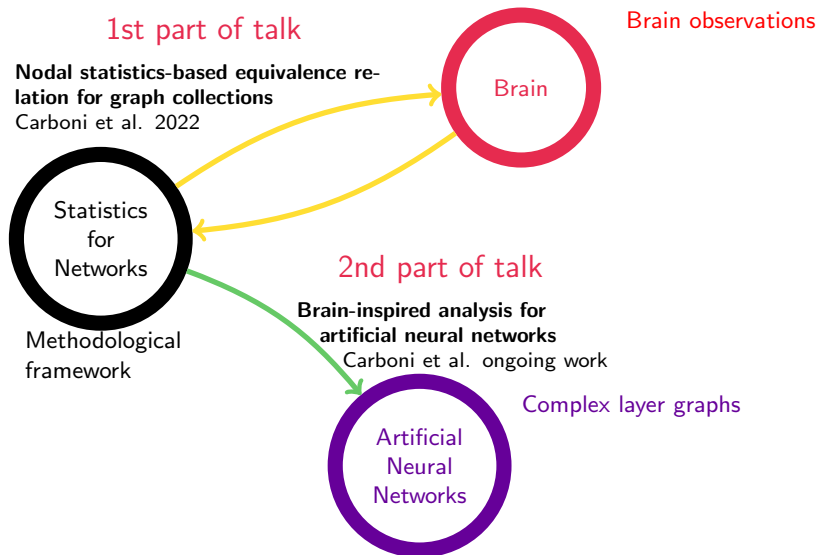
Context



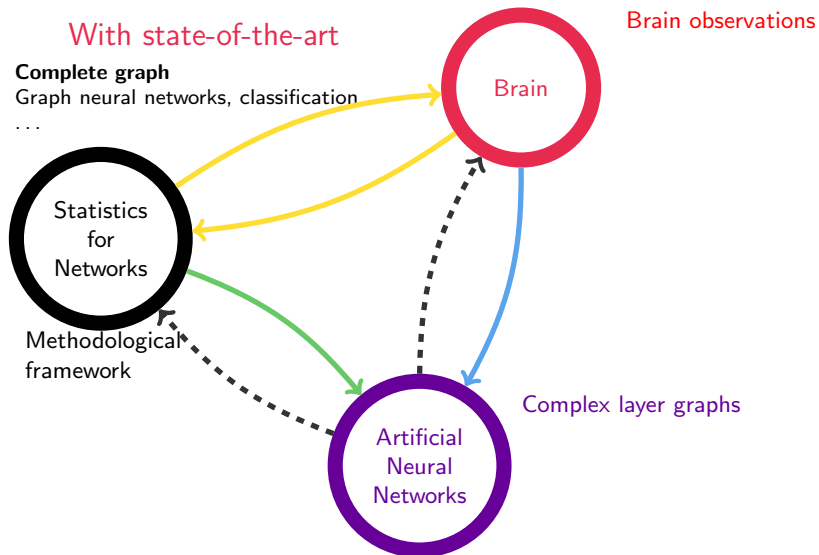
Context



Context



Context

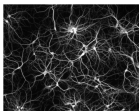


The brain is both a structural and functional network

Microscale

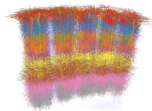


[Cajal, 1890]



[Yuste, 2015]

Mesoscale



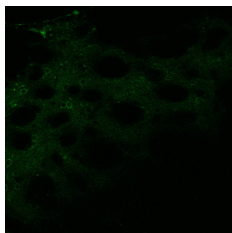
[Oberlaender *et al.*, 2013]

Macroscale



Neuroimaging

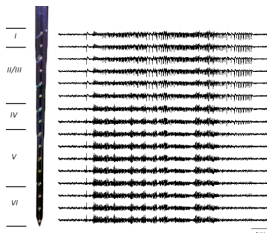
Optical imagery



[Badreddine PhD]

Sophie Achard

Microelectrodes



Thanks to Florian Studer

Graphing the brain

fMRI/EEG/MEG



[De Vico Fallani *et al.*, 2016]

7 October 2026

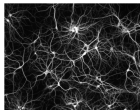
3 / 35

The brain is both a structural and functional network

Microscale

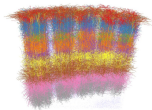


[Cajal, 1890]



[Yuste, 2015]

Mesoscale

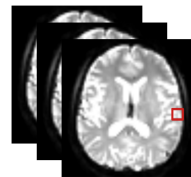
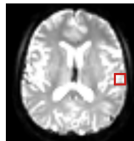


[Oberlaender *et al.*, 2013]

Macroscale



functional MRI



time

Activities of
neurons



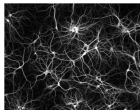
small variations
on signal

The brain is both a structural and functional network

Microscale

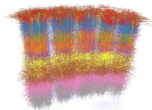


[Cajal, 1890]



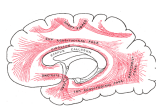
[Yuste, 2015]

Mesoscale

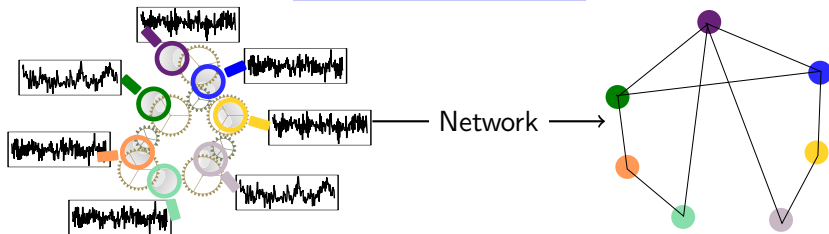


[Oberlaender *et al.*, 2013]

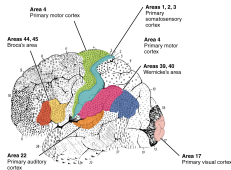
Macroscale



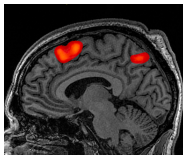
Mathematical modeling



The brain is both a structural and functional network



Regional view

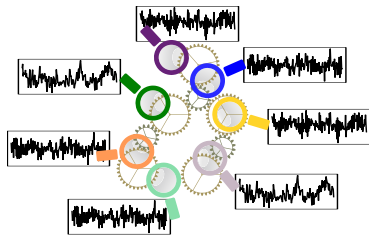


Neuroimaging

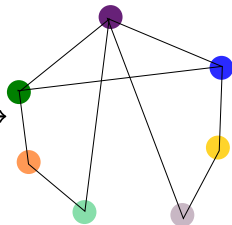


Connectivist view

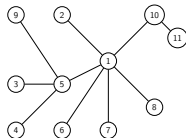
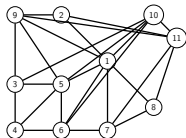
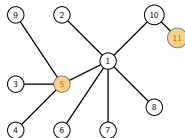
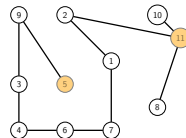
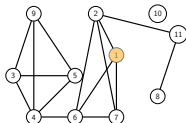
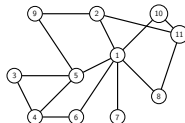
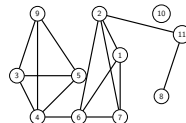
Mathematical modeling



Network



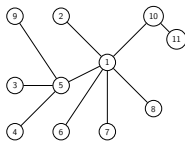
Usual graph statistics



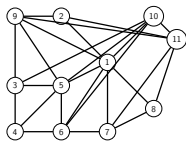
[Latora et al. 2001] [Bullmore et al. 2009]
[Csárdi et al. 2006]

Usual graph statistics

Cost

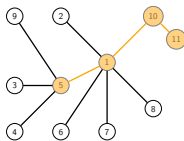


Low cost

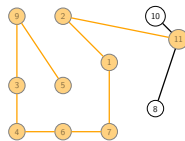


High cost

Efficiency

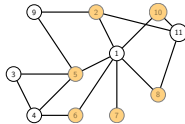


High efficiency

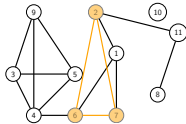


Low efficiency

Clustering

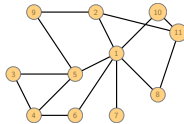


Low clust

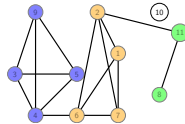


High clust

Modularity



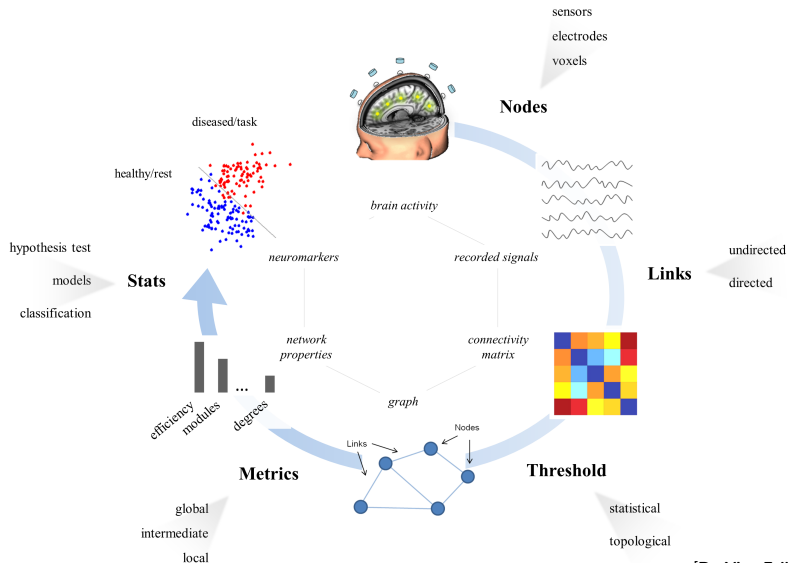
Low modularity



High modularity

[Latora et al. 2001] [Bullmore et al. 2009]
[Csárdi et al. 2006]

Exploring the brain using networks analysis: pipeline



[De Vico Fallani et al. 2014]

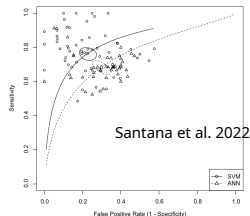
State of the art results for classification

brain regions defined with regions using DictLearn, connectivity matrices parametrized by their tangent-space representation, and an l2-regularized logistic regression as a classifier

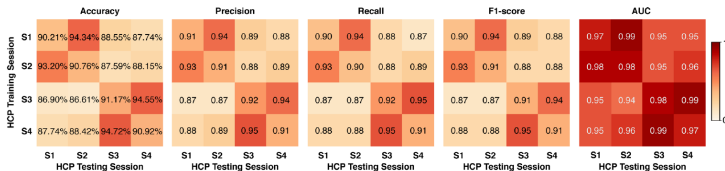
Accuracy	COBRE	ADNIDOD	ADNI	ABIDE	ACPI
5 th percentile	75.5%	69.9%	57.8%	66%	42.5%
Median	86.2%	79.5%	72.5%	71.1%	55.4%
95 th percentile	95%	90.6%	84.5%	75.6%	68.7%

Dadi et al. 2019

		SVM	Random Forest	MLP	BrainNetCNN	GAT	GraphSAGE	PR-GNN	BrainGNN
Biopoint	Accuracy (%)	62.80(4.92) *	68.60(3.58)	58.80(1.79)	75.20(3.49)	77.40(3.51)	78.60(5.90)	77.10(8.71)	79.80(3.63) ^c
	F1 (%)	60.08(3.91)	63.97(4.95)	55.25(9.49)	65.58(14.48)	75.08(5.19)	75.55(7.03)	75.20(7.01)	75.80(6.03)
	Recall (%)	60.20(4.49)	71.11(8.12)	61.00(4.85)	66.20(10.85)	71.60(6.07)	75.20(6.46)	78.26(10.28)	72.60(5.64)
	Precision (%)	60.00(3.81)	67.80(5.36)	53.40(12.52)	65.60(17.95)	79.40(8.02)	76.20(8.11)	76.50(14.32)	79.60(8.59)
	Parameter (k) ^b	3	3	138	1438	16	6	6	41
HCP	Accuracy (%)	90.00(8.20)	90.20(4.15)	67.20(34.40)	90.60(4.04)	78.60(10.45)	89.80(12.51)	91.20(8.28)	94.40(4.04)* ^d
	F1 (%)	90.20(5.81)	90.14(5.55)	63.49(41.80)	90.96(3.50)	77.00(11.58)	88.60(13.19)	91.09(8.35)	94.34(3.27)*
	Recall (%)	89.57(8.04)	90.06(7.35)	67.97(41.66)	91.12(4.13)	78.60(10.45)	89.43(12.43)	91.00(8.95)	94.29(3.73)*
	Precision (%)	90.85(9.35)	90.22(4.77)	62.97(42.47)	90.81(3.27)	91.20(3.32)	87.80(14.02)	91.14(8.52)	94.40(3.59)*
	Parameter (k)	36	36	713	4547	34	12	12	96



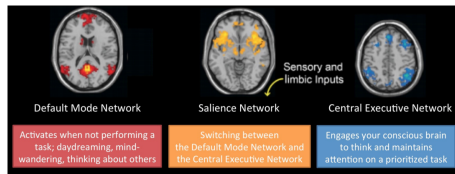
Li et al. 2021



Rvati et al. 2023

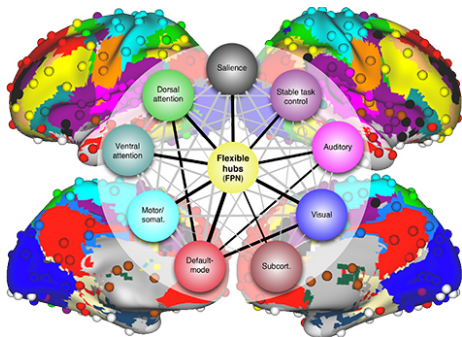
Graphing the brain

Multiple networks in the cognitive brain activity



Large-scale brain networks in cognition: emerging methods and principles

Steven L. Bressler¹ and Vinod Menon²



nature
neuroscience

Article | Published: 28 July 2013

Multi-task connectivity reveals flexible hubs for adaptive task control

Michael W Cole , Jeremy R Reynolds, Jonathan D Power, Grega Repovs, Alan Anticevic & Todd S Braver

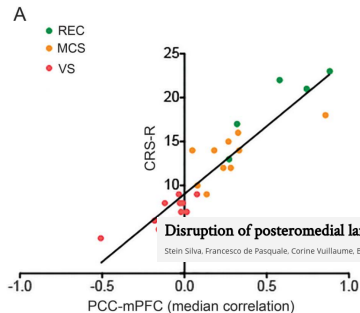
Pathologies and brain connectivity

The hubs of the human connectome are generally implicated in the anatomy of brain disorders

Nicolas A. Crossley,¹ Andrea Mechelli,¹ Jessica Scott,¹ Francesco Carletti,¹ Peter T. Fox,² Philip McGuire^{1,*} and Edward T. Bullmore^{1,4,5,*}

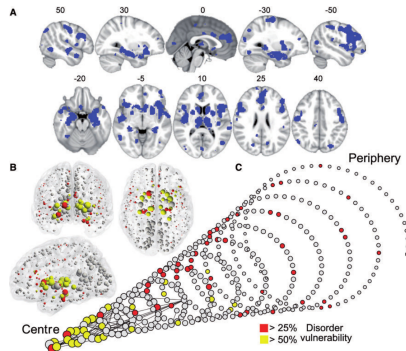
Neurodegenerative Diseases Target Large-Scale Human Brain Networks

William W. Seeley,^{1,*} Richard K. Crawford,¹ Juan Zhou,¹ Bruce L. Miller,¹ and Michael D. Greicius²



Disruption of posteromedial large-scale neural communication predicts recovery from coma

Stein Silva, Francesco de Pasquale, Corine Vuillaume, Beatrice Riu, Isabelle Loubinoux, Thomas Geeraerts, Thierry Seguin, Vincent Bounes, Olivier Fourcade, Jean-François Demonet and Patrice Péran



The Role of Intrinsic Brain Functional Connectivity in Vulnerability and Resilience to Bipolar Disorder

Gaëlle E. Doucet, Ph.D., Danielle S. Bassett, Ph.D., Nalin Yao, Ph.D., David C. Glahn, Ph.D., Sophia Frangou, M.D., Ph.D.,

Outline of the talk

1st Part

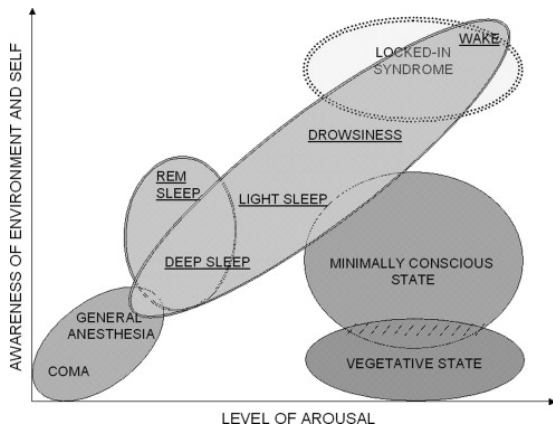
Role of hubs in consciousness disorders

2nd Part

Brain-inspired functional connectivity analysis of Artificial Neural Networks

Brain networks for exploring disorders of consciousness

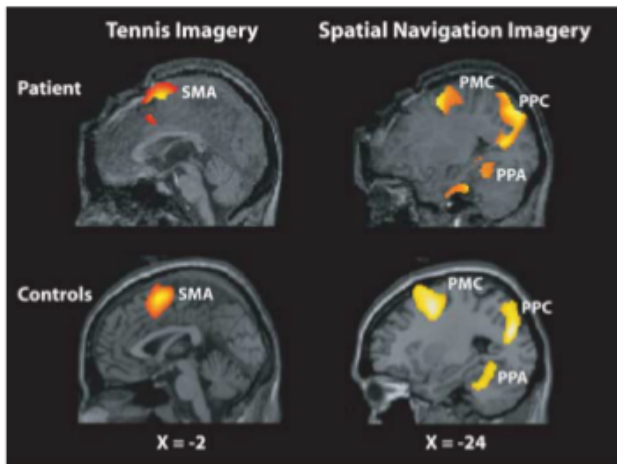
Following Plum and Posner (1983), consciousness has two dimensions: **wakefulness** (also called arousal) and **awareness**.



[Laureys *et al.* 2007]

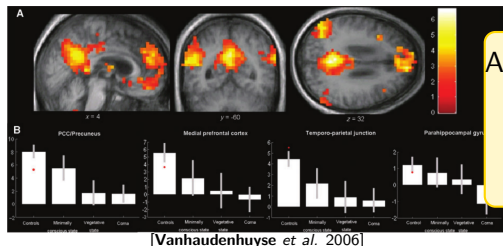
Detecting awareness using fMRI

Using Tennis Imagery to detect awareness for patient with traumatic brain injury.



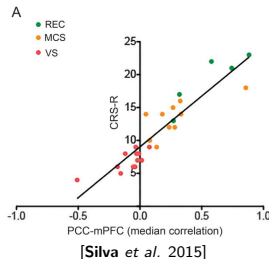
[Owen *et al.* 2006]

Alterations of connectivity in consciousness disorders

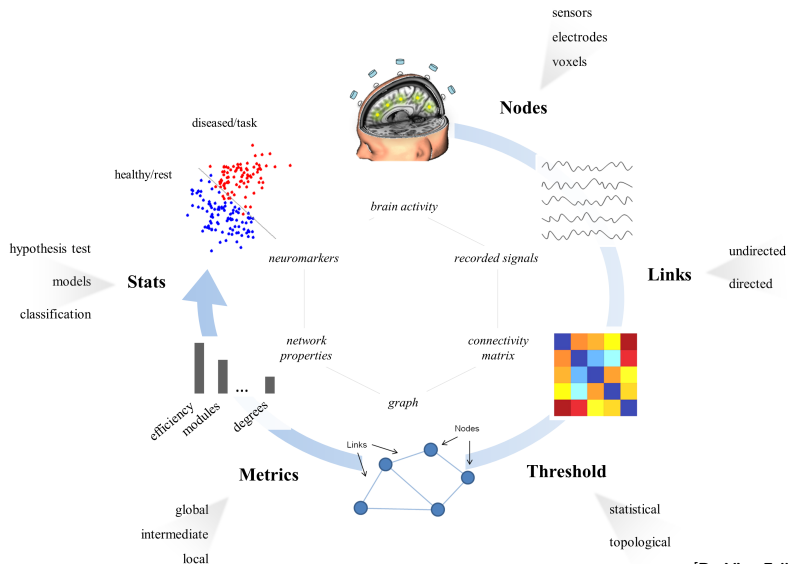


Alterations of Default Mode Network
ICA method for fMRI data
High variance within groups
of patients

Alterations of one connection
fMRI data
CRS = coma recovery state

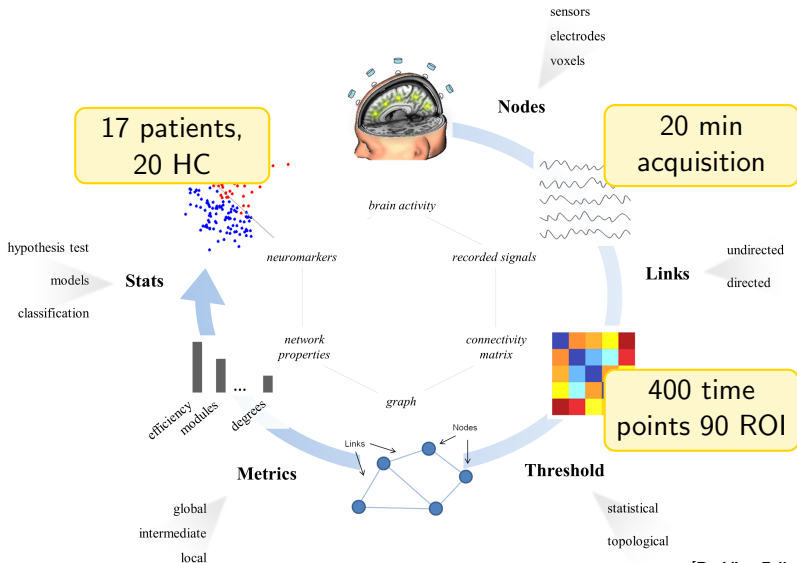


Pipeline



[De Vico Fallani et al.2014]

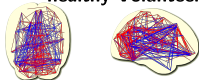
Pipeline



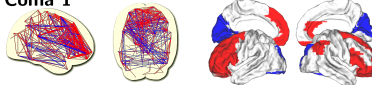
[De Vico Fallani *et al.* 2014]

Examples of connectivity graphs

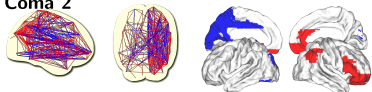
healthy Volunteers



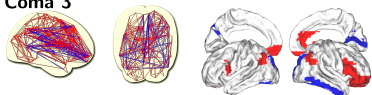
Coma 1



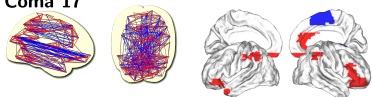
Coma 2



Coma 3



Coma 17



■ Significant decrease

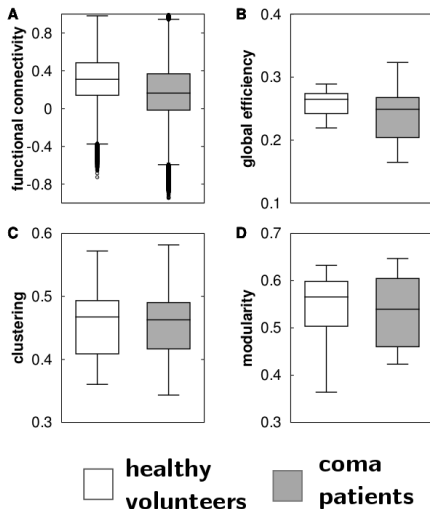
■ Significant increase

Short-range connections

long-range connections

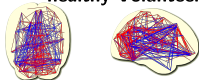
Results: global connectivity and network topology

No significant difference on global measure of functional connectivity

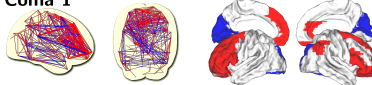


Examples of connectivity graphs

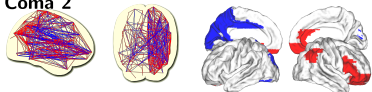
healthy Volunteers



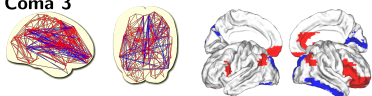
Coma 1



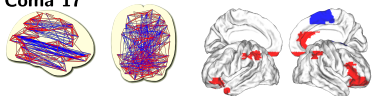
Coma 2



Coma 3



Coma 17



■ Significant decrease

■ Significant increase

Short-range connections

long-range connections

Nodal statistics-based structural pattern on single graph

Nodal statistics-based structural pattern

Definition

We consider undirected unweighed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ and refer to a nodal statistics $s : \mathcal{V} \rightarrow s(\mathcal{V})$ any function of the adjacency matrix.

The **equivalence relation** $\sim_s$ **associated with a nodal statistics** s , on the nodes set $\mathcal{V}$ of a graph is:

$$v \sim_s u \iff s(u) = s(v).$$

The **equivalent relation associated with any collection of statistics** $\mathcal{S} = \{s_i\}_{i=1,\dots,n}$, is defined as:

$$a \sim_{\mathcal{S}} b \iff a \sim_{s_1} b, a \sim_{s_2} b, \dots, a \sim_{s_n} b.$$

When the statistics is continuous, for some small ε

$$v \sim_s u \iff |s(u) - s(v)| \leq \varepsilon$$

Nodal statistics-based structural pattern on single graph

Structural Pattern

Definition

Let us define P the induced partition on $\mathcal{V}$ given $\mathcal{S}$,

$$P_{\mathcal{S}} = \frac{\mathcal{V}}{\sim_{\mathcal{S}}} = \{[a], \quad \forall a \in \mathcal{V}\}$$

defines the **structural pattern** of $\mathcal{G}$ associated with the statistics collection $\mathcal{S}$.

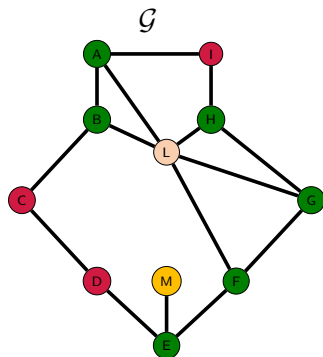
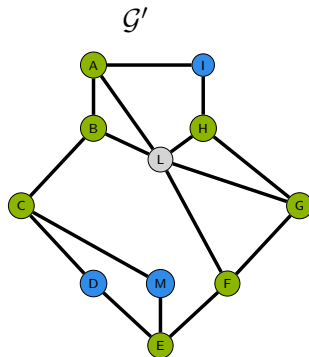
Node role

Definition

The class of equivalence $[a] = \{b \in \mathcal{V} \mid a \sim_s b \iff s(a) = s(b)\}$ corresponds to **node role**.

Illustrations with degree

Let $\mathcal{G}, \mathcal{G}'$ be two graphs having same vertices $\mathcal{V}$ and let $\mathcal{S}$ be the statistics degree whose associated partitions are $P_{\mathcal{S}}, P'_{\mathcal{S}}$ on $\mathcal{G}, \mathcal{G}'$ respectively.

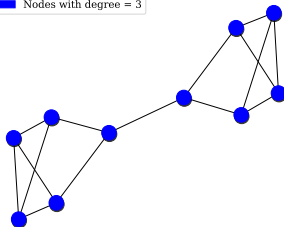


$$P_{\mathcal{S}} = [ABEFGH], [L], [CDI], [M] \quad P'_{\mathcal{S}} = [ABCEFGH], [L], [DIM]$$

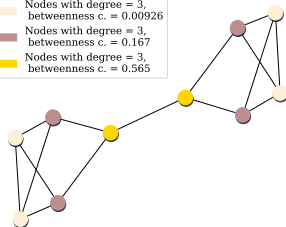
However degree may be not sufficient

Depending on the graph structure, one statistic may be not sufficient to capture important features of graph.

Nodes with degree = 3



Nodes with degree = 3,
betweenness $c.$ = 0.00926
Nodes with degree = 3,
betweenness $c.$ = 0.167
Nodes with degree = 3,
betweenness $c.$ = 0.565



Comparisons of statistics induced by graph structures

Orthogonal statistics for heterogeneity evaluation of a collection elements

Two nodal statistics are said to be perfectly orthogonal if their union-associated equivalent relation induces the trivial partition: all nodes belong to a single element set.

Definition

Let $\mathcal{G}$ be a graph having $\mathcal{V}$ vertices and let $\mathcal{S}$ be the statistics whose associated partitions are $P_{\mathcal{S}}$ on $\mathcal{G}$.

$$O_{\mathcal{G}}(\mathcal{S}) = \frac{|\{v \in \mathcal{V} \text{ s.t. } \#[v]_{\sim_{\mathcal{S}}} \neq 1\}|}{|\mathcal{V}|}$$

$O_{\mathcal{G}}(\mathcal{S})$ is the ratio between the number of nodes in non-trivial classes and the total number of vertices and corresponds to an orthogonality score. By definition, $\mathcal{S}$ is perfectly orthogonal if and only if $O_{\mathcal{G}}(\mathcal{S}) = 0$.

Structural patterns for graph collections characterization

Correspondence structural pattern score

Definition

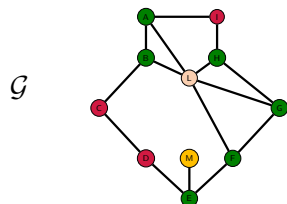
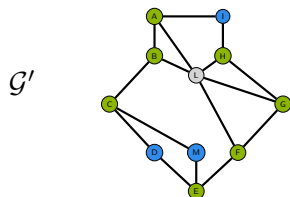
Let $\mathcal{G}, \mathcal{G}'$ be two graphs having same vertices $\mathcal{V}$ and let $\mathcal{S}$ be a statistics collection whose associated partitions are $P_{\mathcal{S}}, P'_{\mathcal{S}}$ on $\mathcal{G}, \mathcal{G}'$ respectively. Given bijective mapping from $P_{\mathcal{S}}, P'_{\mathcal{S}}$ to an initial segment of the natural numbers as enumerations, let $c(v_i), c'(v_i)$ be the enumeration of the classes of v_i , the correspondence structural pattern score between $\mathcal{G}, \mathcal{G}'$ is defined as:

$$C(\mathcal{G}, \mathcal{G}') = \max_{\pi \in \Pi} \frac{1}{|\mathcal{V}|} \sum_{i=1}^{|\mathcal{V}|} \mathcal{X}(\pi(c(v_i)) = c'(v_i))$$

where Π is the set of all coupling between the elements in $P_{\mathcal{S}}$ and the elements in $P'_{\mathcal{S}}$ and $\mathcal{X}$ is the indicator function.

Illustrations

Let $\mathcal{G}, \mathcal{G}'$ be two graphs having same vertices $\mathcal{V}$.



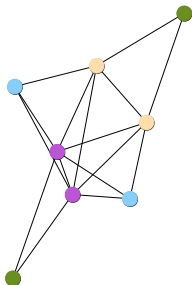
		$\mathcal{G}$			
		L	C D I	A B E F G H	M
$\mathcal{G}'$	L	1			
	D I M		2		1
	A B C E F G H		1	6	

$$C(\mathcal{G}, \mathcal{G}') = \frac{1+2+6}{11} = 0.81$$

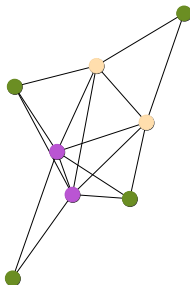
Illustrations

One statistics is enough informative

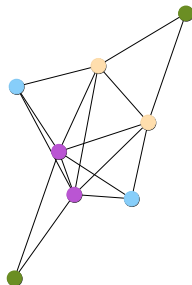
Degree
 $\hat{PC}$: 0.73855



Betweenness
 $\hat{PC}$: 0.56959

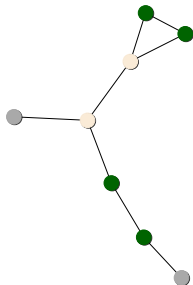


Degree and betweenness
 $\hat{PC}$: 0.73855
Correspondence structural patterns: 0.75
Orthogonality in G: 1.0

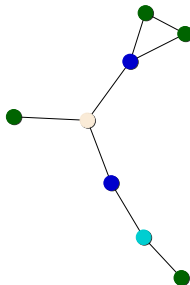


Two nodal statistics are more informative

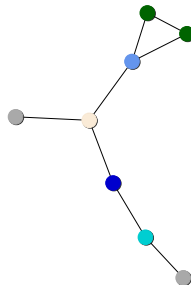
Degree
 $\hat{PC}$: 0.56959



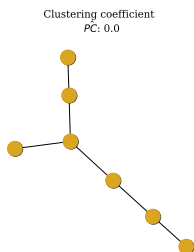
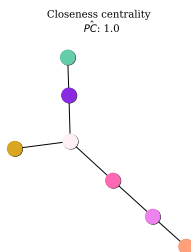
Betweenness
 $\hat{PC}$: 0.63495



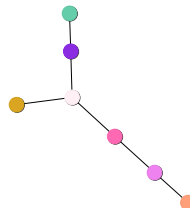
Degree and betweenness
 $\hat{PC}$: 0.86927
Correspondence structural patterns: 0.5
Orthogonality in G: 0.5



Two perfectly orthogonal nodal statistics



$PC: 1.0$
Correspondence structural patterns: 0.1428571
Orthogonality in G: 0.0

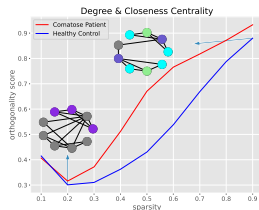


Results of fMRI resting state datasets on healthy volunteers and patients

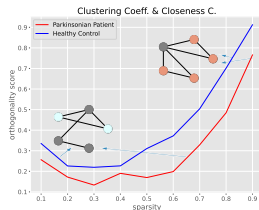
Freely available datasets. Do not hesitate to use them!

Provider	Subjects Total	Scanning Parameters				Age range years
		B0	TR	#Vol	Frequency Band	
ChuStr <i>Achard et al. 2012</i>	HC(20)-CO(17)	1.5T	3000 ms	405	0.042-0.084 Hz	25-45, 21-82
HCP <i>Termenon et al. 2016</i>	HC(100×2)	3T	720 ms	1200	0.043-0.087 Hz	20-43
iShare <i>Tsuchida et al. 2017</i>	HC(1814)	3T	850 ms	1046	0.037-0.074 Hz	18-35
Gin-Chuga <i>Ramirez et al. 2019</i>	HC(11)-PD(13)	3T	1000 ms	500	0.031-0.063 Hz	46-70, 51-70

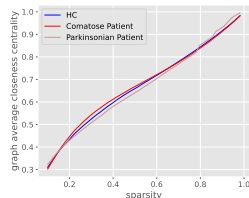
Results of orthogonality scores on real datasets



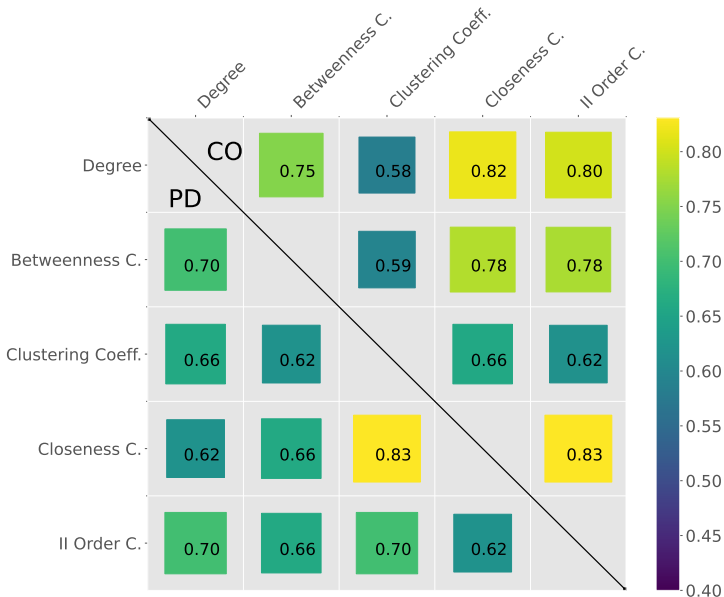
(a) Orthogonality centroids



(b) Mean graph average



Illustrations of classification on orthogonality curves



Outline of the talk

1st Part

Role of hubs in consciousness disorders

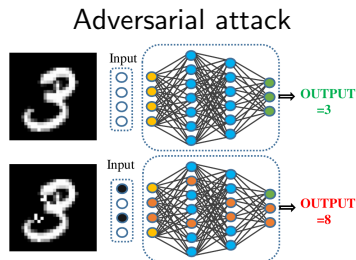
2nd Part

Brain-inspired functional connectivity analysis of Artificial Neural Networks

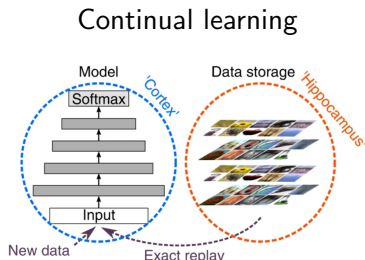
Statistics for networks applied to ANNs

Question: Is it possible to identify results of classification on neural networks?

Examples:



[Yu et al. 2018]



[van de Ven et al. 2020]

Focus on catastrophic forgetting: How to characterize the effect of catastrophic forgetting on the architecture of neural networks?

Post-doc Dwight Ngwaigwe & PhD Lucrezia Carboni & PhD Marion Mainsant

Catastrophic forgetting: our context

Continual learning with face emotion recognition

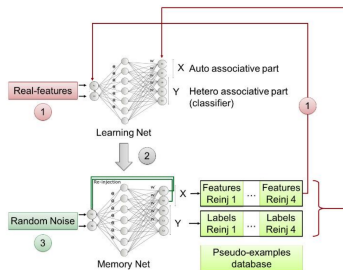
PhD work of M. Mainsant with M. Mermillod and M. Reyboz

Continual learning settings



7 different tasks

Dream Net

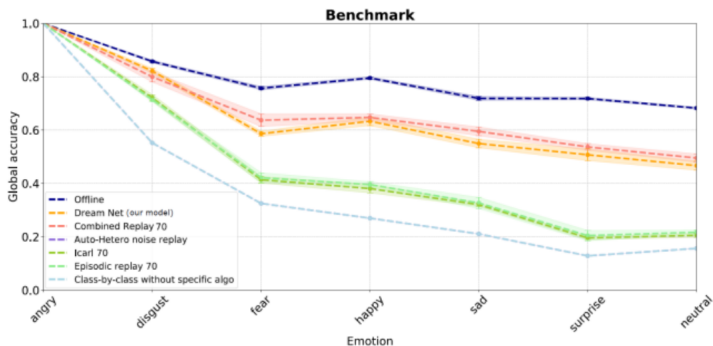


[Mainsant *et al.* 2021]

Catastrophic forgetting: our context

Continual learning with face emotion recognition

PhD work of M. Mainsant with M. Mermillod and M. Reyboz

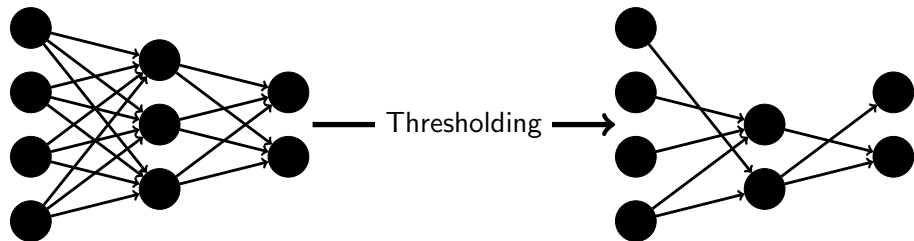


Are we able to quantify why Dream Net performs well?

Our proposed pipeline

Objective: Extend graph-statistics analysis method to ANNs to characterize flow of information in a NN model

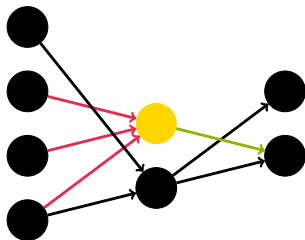
- Step 1: Graph modelling of ANNs



Our proposed pipeline

Objective: Extend graph-statistics analysis method to ANNs to characterize flow of information in a NN model

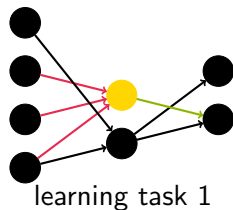
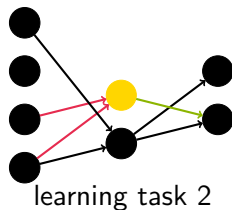
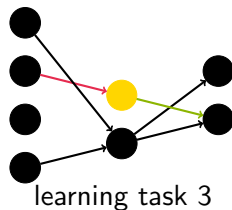
- Step 1: Graph modelling of ANNs
- Step 2: Node role **in** and **out** degree



Our proposed pipeline

Objective: Extend graph-statistics analysis method to ANNs to characterize flow of information in a NN model

- Step 1: Graph modelling of ANNs
- Step 2: Node role **in** and **out** degree
- Step 3: Characterisation - graph features extraction



Graph Features vector

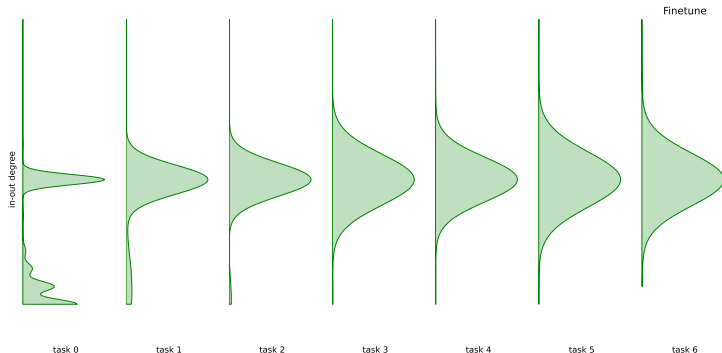
in-out
degree ● =2

● =1

● =0

Graph features vectors as distributions

- Finetune strategie

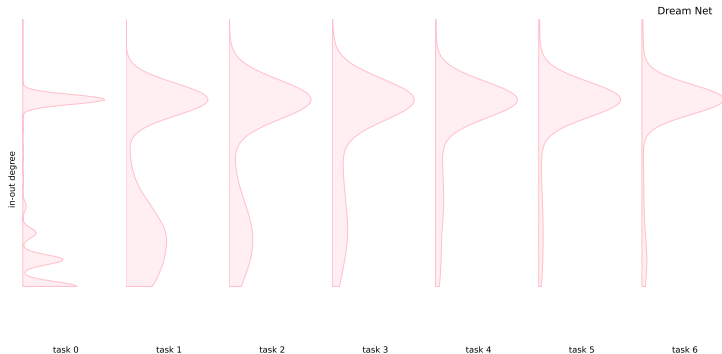


Focus on the tail of the distributions

Keep only 1st and 3rd quartiles of the distributions

Graph features vectors as distributions

- Dream Net strategie

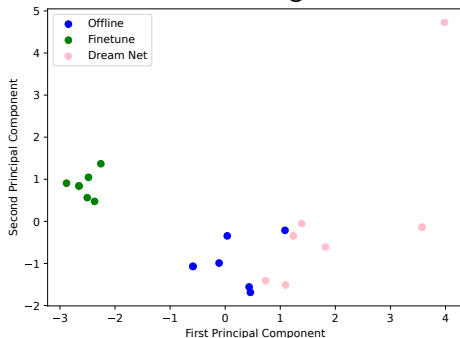


Focus on the tail of the distributions

Keep only 1st and 3rd quartiles of the distributions

Results on real datasets

Consensus score clustering-labels = 0.965



- **Offline**: “null model” re-learning every previously seen tasks
- **Finetune**: online learning with forgetting
- **Dream Net**: online learning without forgetting

in-out degree graph feature is able to discriminate between different strategies taking into account forgetting

Conclusion and future works

Conclusion

- New framework for comparison of spatio-temporal models
- Description at nodal level
- Application to ANNs

Future work

- Derive properties for graph statistics
- Build a statistical test
- Formalise the ANNs application



arxiv.org/abs/2210.01053



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**Lucrezia Carboni
& Michel Dojat**

Verónica Muñoz Ramírez

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Dwight Nwaigwe

Marion Mainsant

Marina Reyboz

Martial Mermillod

Marc Joliot

Raphaël Bayle

Ana Marques

Elena Moro

Lydia Oujamaa

Argheesh Bhanot

Paul Sitoleux

Francesco Pappone

Simone Chiominto

Youssef Saad

NIPCs & Statifiers !

